DECOHERENCE-FREE ENTROPIC GRAVITY: MODEL & EXPERIMENTAL
TESTS
AN HONORS THESIS
SUBMITTED ON THE 4TH DAY OF MAY, 2020
TO THE DEPARTMENT OF PHYSICS AND ENGINEERING PHYSICS
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
OF THE HONORS PROGRAM
OF NEWCOMB-TULANE COLLEGE
TULANE UNIVERSITY
FOR THE DEGREE OF
BACHELOR OF SCIENCES
WITH HONORS IN PHYSICS
BY

Alex Schimmoller

APPROVED:
Dr. Denys Bondar
Director of Thesis
Dr. Fred Wietfeldt
Second Reader
Dr. John Howard
Third Reader
Quantum mechanics and general relativity are two of physics' most comprehensive descriptions of the universe. However, a leading theory of quantum gravity is yet to be established. Several theories which probe at the fundamental assumptions of gravitation have been proposed, including Erik Verlinde's theory of entropic gravity, presuming that gravity is not a fundamental force, but rather emerges as a consequence of the second law of thermodynamics. Some have ruled out this interpretation on grounds that entropic forces are by nature noisy and entropic gravity would therefore display far more decoherence than is observed. This thesis addresses such criticism by modeling gravity as an open quantum system, arrive at a master equation which can mimic conservative gravity with arbitrarily high precision and low decoherence, then compare our theoretical results to data from ultra-cold neutron experiments.
Acknowledgements

First and foremost, I wish to express my deepest appreciation to my research advisor Dr. Denys Bondar. This project would not have been possible without his patience, guidance, and charismatic involvement at every step. It has been a pleasure learning from him.

I am also extremely grateful to my current and former committee members Dr. Fred Wietfeldt, Dr. John Louis Howard and Dr. Wayne Reed. Each has provided thoughtful feedback and criticism over the past year which has positively shaped this project’s development.

Furthermore, special thanks to the Tulane Honors Summer Research Program. The depth and rigor of this project could not have been achieved without funding and resources made available through this program.

Finally, my sincerest thanks to my parents Jim and Jayne and my brother Nick, along with Peter Michalakes, Ethan Faries and Mallory Johnston, all of whom have provided unwavering support over the course of this research.
Table of Contents

I. Introduction ................................................................................................................. 1
II. A Model of Entropic Gravity Acting Near Earth’s Surface ..................... 2
III. Modeling the QBounce Experiment ................................................................. 6
IV. Simulating the QBounce Experiment ................................................................. 8
V. Conclusions and Further Directions ............................................................... 9
VI. Bibliography ......................................................................................................... 11
VII. Appendices ............................................................................................................. 13
List of Figures

1. Transmission probability for varying oscillation frequency when the oscillation strength equals 2.05 mm/s. ................................................................. 10

2. Transmission probability for varying oscillation strength when the boundary frequency $\omega = \omega_{02} = \frac{E_2-E_0}{\hbar}$ ...................................................... 10

3. Transmission probability for varying oscillation strength when the boundary frequency $\omega = \omega_{03} = \frac{E_3-E_0}{\hbar}$ ...................................................... 11
I. Introduction

To date, a leading theory of quantum gravity is yet to be developed, let alone tested experimentally. One potential resolution to the quantum gravity problem is the theory of entropic gravity, challenging the assumption that gravity is a conservative force, that is, one dictated by a potential. Entropic gravity instead postulates that gravity is an entropic force, or one that points in the direction of maximum entropy.

The mathematical equation for entropic forces follows simply from the first law of thermodynamics which equates heat supplied to a system $\delta Q$ to the change in the system’s internal energy $dU$ plus work done unto the system $\delta W$, or

$$\delta Q = dU + \delta W. \quad (1)$$

After substituting in the thermodynamic definition of entropy $dS := \frac{dQ}{T}$ for systems at constant temperature $T$, the previous equation can be rewritten as

$$TdS = dU + \delta W. \quad (2)$$

When the internal energy is constant, $dU = 0$ and equation (2) reduces to

$$TdS = \delta W = F_E \cdot dr. \quad (3)$$

Say this system operates in one dimension. Then $F_E \cdot dr = F_E dr$. Furthermore, say the entropy $S$ is a function of $r$ only. Then the entropic force $F_E$ on the system is given by

$$F_E(r) = T \frac{dS}{dr} = TS'(r). \quad (4)$$

That is to say, at a constant temperature $T$, entropic forces are directly proportional to the entropy gradient $S'(r)$.

While gravity appears to be conservative, Erik Verlinde has garnered much attention for his proposal that gravity is entropic in nature [13]. He takes insights from Jacob Bekenstein [2] who argued that a particle of mass $m$ held by a string just outside a black hole will effectively be absorbed once the particle is held within one Compton wavelength,

$$\Delta x = \frac{h}{mc}. \quad (6)$$
of the event horizon. Here, $h$ is Planck’s constant and $c$ is the speed of light. At this point, it is unknown whether the particle still exists or has been destroyed inside the black hole. So, the particle has gone from being in a pure ”exists” state to either an ”exists” or ”destroyed” state, both of which have equal probabilities of 1/2. Using Boltzmann’s formula for entropy

$$S = k_b \ln(\Omega),$$

(7)

where $k_b$ is Boltzmann’s constant and $\Omega$ is the number of available states, it can at minimum be said that $\Omega$ has increased from 1 to 2. Hence, the black hole’s entropy has increased by $k_b \ln(2) - k_b \ln(1)$, giving a change in entropy

$$\Delta S = k_b \ln(2).$$

(8)

So a black hole, essentially an extreme concentration of mass in the universe, has an entropy tied to it. If this idea is generalized to lesser concentrations of mass, then they should also have an associated entropy. Verlinde postulates from Bekenstein’s argument that the change in entropy $\Delta S = 2\pi k_b$ when a particle is one Compton wavelength (6) from a holographic screen in space. Assuming that entropy is linear with respect to $\Delta x$, he generalizes to say that

$$\Delta S = 2\pi k_b \frac{mc}{\hbar} \Delta x.$$  

(9)

Rewriting the entropic force equation (5) as

$$F \Delta x = T \Delta S$$

(10)

and employing Unruh’s formula,

$$k_b T = \frac{1}{2\pi} \frac{\hbar a}{c},$$

(11)

where $a$ is acceleration, he arrives at the familiar force equation, $F = ma$.

Verlinde’s theory has undergone scrutiny, especially over his invocation of holographic screens and the Unruh formula [8] [9] [10] [14], although these criticisms also seem to acknowledge some connection between thermodynamics and gravity [6] [12].

Perhaps gravity is entropic, but the right interpretation hasn’t come along yet. There is still one prevailing criticism of all entropic gravity theories which deserves consideration [8] [9] [10] [14]: while a full quantum theory of entropic gravity is yet to be established, some have ruled it out on grounds that entropic forces are by nature noisy and entropic
gravity would break quantum coherence. In particular, it is argued in [14] that if gravity is an entropic force, then it can be modeled as an environment in an open quantum system. Brownian motion is not observed for small objects inside the environment, so these small objects must be very strongly coupled to the gravity environment. But if objects are strongly coupled, then they must exhibit ample wavefunction collapse and quantum decoherence. However, such decoherence is not observed in cold neutron experiments [11] on Earth’s surface, as is pointed out in [8] [9]. Thus, entropic gravity cannot be true. Taking this criticism of entropic gravity seriously, we treat gravity near Earth’s surface as an open quantum system whose average dynamics match that of conservative gravity and show that such a system can display both strong coupling and low decoherence.

II. A Model of Entropic Gravity Acting Near Earth’s Surface

In this section, we develop a near-Earth model of entropic gravity acting on small objects. Consider a particle of mass $m$ a small distance $x$ above Earth’s surface in free-fall. In the classical case, the particle’s dynamics are dictated by the equations

$$\frac{d}{dt} x = \frac{1}{m} p,$$

$$\frac{d}{dt} p = -mg,$$  \hspace{1cm} (12)

where $p$ is the particle’s momentum, $g$ is the gravitational acceleration near Earth’s surface and the derivatives are with respect to time $t$. Combined, these two equations are a formulation of Newton’s second law, stating that the force $F = m \frac{d^2 x}{dt^2} = -mg$. In the quantum regime, however, these dynamics must be recast in the language of operators and expectation values. This is accomplished by invoking the Ehrenfest theorems which translate classical physical parameters $x$ and $p$ into expectation values $\langle \hat{x} \rangle$ and $\langle \hat{p} \rangle$:

$$\frac{d}{dt} \langle \hat{x} \rangle = \frac{1}{m} \langle \hat{p} \rangle,$$

$$\frac{d}{dt} \langle \hat{p} \rangle = -mg.$$  \hspace{1cm} (14)

Notice that Newton’s second law describes the system’s dynamics precisely whereas the Ehrenfest theorems describe average dynamics. This latter formulation not only reflects actual experimental analysis, but also allows one to formulate a quantum mechanical master equation satisfying the above equations.
To begin, we define a conservative master equation for free-fall. An object of mass $m$ a small distance $x$ above Earth’s surface and experiencing a linear gravitational potential $gx$ can be described using the Liouville equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \hat{\rho}^2 \frac{\hat{p}^2}{2m} + mg\hat{x}, \hat{\rho} \right],$$

(16)

where $\hat{\rho}$ is the system’s density matrix and $\hbar$ is half-Planck’s constant. Here, the system’s Hamiltonian consists of a kinetic energy term $\frac{\hat{p}^2}{2m}$ plus a potential energy term $mg\hat{x}$. With expectation values for an arbitrary operator $\hat{O}$ given by

$$\langle \hat{O} \rangle = \text{Tr}(\hat{O}\hat{\rho}),$$

(17)

it can easily be shown that this master equation satisfies the free-fall Ehrenfest theorems (14), (15). Equation (16) will thus serve as the conservative model for free-fall.

Equation (16), is not the only master equation which has these dynamics, though. In fact, within the language of open quantum systems, there are an infinite number of master equations which satisfy the above Ehrenfest theorems. It has been shown through quantum reservoir engineering [15] that the general Ehrenfest theorems

$$\frac{d}{dt} \langle \hat{x} \rangle = \langle G(\hat{p}) \rangle,$$

(18)

$$\frac{d}{dt} \langle \hat{p} \rangle = \langle F(\hat{x}) \rangle$$

(19)

can be satisfied by coupling a closed system with the usual Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x})$ to a series of environments. We take advantage of this fact to model gravity as an environment in an open quantum system.

In the simplest case, gravity can be treated as a single dissipative environment and the free-fall dynamics given by (14) and (15) are satisfied by the Lindblad equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \hat{\rho}^2 \frac{\hat{p}^2}{2m\hbar} \hat{p} + \mathcal{D}(\hat{\rho}) \right],$$

(20)

$$\mathcal{D}(\hat{\rho}) = \frac{mgx_0 \mathcal{F}}{\hbar} \left\{ \exp \left( -\frac{i\hat{x}}{x_0 \mathcal{F}} \right) \hat{\rho} \exp \left( +\frac{i\hat{x}}{x_0 \mathcal{F}} \right) - \hat{\rho} \right\},$$

(21)

where

$$x_0 = \left( \frac{\hbar^2}{2m^2g} \right)^{1/3}$$

(22)
is a scaling factor and $\mathcal{S}$ is a unitless, non-negative free parameter in the model. We postulate that $\mathcal{S}$ is constant for all objects residing near Earth’s surface. Note that the Hamiltonian in this case only contains the kinetic energy term $\hat{p}^2/2m$, and the potential energy term $mg\hat{x}$ has effectively been absorbed into the dissipator $\mathcal{D}(\hat{\rho})$. Equation (20) will serve as the model for entropic gravity acting near Earth’s surface.

At first glance, it is not readily apparent how equations (16) and (20) can produce the same dynamics. In particular, it may seem entirely unclear how the dissipative term $\mathcal{D}(\hat{\rho})$ can mimic a linear gravitational potential. Taking the Hausdorff expansion of $\mathcal{D}(\hat{\rho})$ in (20) gives

$$
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \frac{\hat{p}^2}{2m} + mg\hat{x}, \hat{\rho} \right] + \frac{mg}{x_0\hbar\mathcal{S}} \left( \hat{\rho} \hat{x} - \frac{1}{2} \hat{x}^2 \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{x}^2 \right) + O\left( \frac{1}{\mathcal{S}^2} \right).
$$

Thus in the limit where $\mathcal{S}$ approaches infinity, terms proportional to $\mathcal{S}^\text{-1}$ or lower powers disappear and equation (23) mimics the conservative master equation (16) with arbitrarily high precision.

Now, the major criticism of entropic gravity we hope to address with the entropic model is that entropic gravity is strongly coupled to small objects and therefore would break quantum coherence. For conservative systems not coupled to any environments, the purity $\text{Tr}(\hat{\rho}^2) = 1$ for all time $t$. That is to say, conservative quantum systems maintain coherence. For dissipative systems, however, the purity is not constant in time and can dip below 1. This is because the quantum environment is effectively measuring the conservative system when the two interact, whereas no such interaction takes place for conservative systems alone. If the proposed entropic model is going to address the decoherence criticism, it must maintain purity.

Rather than analyzing the purity directly, consider how it changes with time:

$$
\frac{d}{dt} \text{Tr}(\hat{\rho}^2) = \text{Tr} \left( \frac{d}{dt} \hat{\rho}^2 \right) = 2 \text{Tr} \left( \hat{\rho} \frac{d\hat{\rho}}{dt} \right).
$$

When the Hausdorff expansion with respect to $\mathcal{S}$ (23) is plugged into equation (25),

$$
\frac{d}{dt} \text{Tr}(\hat{\rho}^2) = -2 \frac{mg}{x_0\hbar\mathcal{S}} \text{Tr} \left( \hat{\rho}^2 \hat{x}^2 - (\hat{\rho} \hat{x})^2 \right) + O\left( \frac{1}{\mathcal{S}^2} \right).
$$
It is shown using the Cauchy-Schwarz inequality in [15] that \( \text{Tr} \left( \hat{\rho}^2 \hat{x}^2 - (\hat{\rho} \hat{x})^2 \right) \geq 0 \). Thus, purity is monotonically decreasing in the asymptotic limit for \( \mathcal{S} \). Furthermore, the purity rate of change approaches zero as \( \mathcal{S} \) approaches infinity. Since we can elect to make \( \mathcal{S} \) arbitrarily large in our model, the original criticism of entropic gravity not maintaining quantum coherence can no longer be considered valid.

It is also insightful to write \( x_0 \) explicitly in the expansion. In particular, plugging (22) into (20) and taking the Huasdorff expansion with respect to mass \( m \) yields

\[
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \frac{\hat{\rho}^2}{2m} + mg\hat{x}, \hat{\rho} \right] + \frac{1}{\mathcal{S}} \left( \frac{2m^5 g^4}{\hbar^5} \right)^{1/3} \left( \hat{x} \hat{\rho} \hat{x} - \frac{1}{2} \hat{x}^2 \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{x}^2 \right) + O \left( m^{7/3} \right).
\]

This indicates that the truncated expression is a valid approximation when \( m \) is small. Performing the purity analysis with respect to \( m \) shows that

\[
\frac{d}{dt} \text{Tr} (\hat{\rho}^2) = -\frac{1}{\mathcal{S}} \left( \frac{16m^5 g^4}{\hbar^5} \right)^{1/3} \text{Tr} (\hat{\rho}^2 \hat{x}^2 - (\hat{\rho} \hat{x})^2) + O \left( m^{7/3} \right).
\]

Thus for constant \( \mathcal{S} \), purity is also maintained in the small-mass limit. For larger masses, purity decay increases and decoherence effects are more prevalent.

### III. Modeling the Qbounce Experiment

Now that free-fall models for conservative and entropic gravity have been established, it is desirable to see how they compare to results of the Qbounce experiment [3]. In this setup, ultra-cold neutrons are prepared in a superposition of the first four energy states according to solutions to the quantum bouncer problem (A13). These neutrons then traverse a horizontal boundary which can oscillate with variable frequency \( \omega \) and oscillation strength \( b \). (To see how \( b \) appears in the master equations, refer to appendix C). The boundary induces Rabi oscillations in the neutrons and changes their state. After passing through this region, neutrons in the ground state are detected. In order to effectively model this experiment, the free-fall master equations (16) and (20) must also account for neutron collisions with the oscillating boundary.
The most effective means of accomplishing this is to implement the boundary condition into the Ehrenfest theorems. For a system with general Hamiltonian
\[ \hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x}), \] (29)
and boundary condition \( \langle x = 0 | \psi \rangle = 0 \), the system’s second Ehrenfest theorem can be corrected with a boundary term:
\[ \frac{d}{dt} \langle \hat{p} \rangle = \left\langle -U'(\hat{x}) \right\rangle + \frac{\hbar^2}{2m} \left( \frac{d}{dx} \langle x | \psi \rangle \right) \Bigg|_{x=0} \left( \frac{d}{dx} \langle \psi | x \rangle \right) \Bigg|_{x=0} \] (30)
\[ = \left\langle -U'(\hat{x}) \right\rangle + \frac{\hbar^2}{4m} \langle \delta''(\hat{x}) \rangle, \] (31)
where \( \delta \) is the Dirac Delta Function, defined as
\[ \int_{-\infty}^{\infty} dx \delta^{(n)}(x - x') f(x) = (-1)^n f^{(n)}(x'). \] (32)
Thus modifying the Hamiltonian \( \hat{H} \) to include the boundary term \( -\frac{\hbar^2}{4m} \delta'(\hat{x}) \) will provide the desired Ehrenfest theorems:
\[ \hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x}) - \frac{\hbar^2}{4m} \delta'(\hat{x}). \] (33)
In order to make the boundary oscillate in this general case, one simply needs to add a sinusoidal term inside of the Dirac delta function:
\[ \hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x}) - \frac{\hbar^2}{4m} \delta'(\hat{x} - a \sin \omega t). \] (34)
Here, \( a \) is the oscillation amplitude and \( \omega \) is oscillation frequency.

In the particular case of conservative gravity, a neutron’s dynamics while inside the QBounce apparatus can be described by the Liouville equation:
\[ \frac{d}{dt} \hat{\rho} = -\frac{i}{\hbar} \left[ \frac{\hat{p}^2}{2m} + mg\dot{x} - \frac{\hbar^2}{4m} \delta'(\hat{x} - a \sin(\omega t)), \hat{\rho} \right]. \] (35)
Note that this equation includes potential energy, kinetic energy and boundary terms inside the commutator. Meanwhile, the entropic case gives
\[ \frac{d}{dt} \hat{\rho} = -\frac{i}{\hbar} \left[ \frac{\hat{p}^2}{2m} - \frac{\hbar^2}{4m} \delta'(\hat{x} - a \sin(\omega t)), \hat{\rho} \right] + \hat{\mathcal{D}}(\hat{\rho}). \] (36)
Here, the kinetic and boundary terms are inside the commutator and \( \mathcal{D}(\hat{\rho}) \) once again serves as the gravity environment. Most importantly, the oscillating boundary does not alter the dissipator in the entropic case.

For simulations of the Qbounce experiment, we take equations (35) and (36) and move them in the reference frame of the oscillating boundary (see Appendix B). After applying the change of variables \( \tilde{x} = x - a\sin(\omega t) \) and translating \( \tilde{x} \to x \), the conservative model’s Liouville equation becomes

\[
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \frac{\hat{p}^2}{2m} + mg\dot{x} - \frac{\hbar^2}{4m} \delta'(\dot{x}) - a\omega \cos(\omega t) \hat{p}, \hat{p} \right],
\]  

(37)

and the entropic Lindblad equation becomes

\[
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \frac{\hat{p}^2}{2m} - \frac{\hbar^2}{4m} \delta'(\dot{x}) - a\omega \cos(\omega t) \hat{p}, \hat{p} \right] + \mathcal{D}(\hat{\rho}).
\]  

(38)

Differentiating with respect to unitless time \( \tau = tmgx_0/\hbar \) yields the unitless conservative Liouville equation

\[
\frac{d\hat{\rho}}{d\tau} = -i \left[ \hat{h} + \hat{\xi} + \hat{\omega}, \hat{\rho} \right],
\]  

(39)

along with the unitless entropic Lindblad equation

\[
\frac{d\hat{\rho}}{d\tau} = -i \left[ \hat{h} + \hat{\omega}, \hat{\rho} \right] + \mathcal{D} \left( \hat{D}\hat{\rho}\hat{D}^\dagger - \hat{\rho} \right).
\]  

(40)

Here, \( \hat{h} \) represents the kinetic energy and boundary terms, \( \hat{\xi} \) gives the potential energy term, \( \hat{\omega} \) accounts for the accelerating frame and \( \hat{D} \) gives the first exponential inside the \( \mathcal{D}(\hat{\rho}) \) term. Matrix elements for these operators can be found in appendix C. Equations (39) and (40) will serve as the scaled master equations for Qbounce simulations.

**IV. Simulating the QBounce Experiment**

Using results from the previous section, we can effectively simulate the QBounce experiments from reference [3]. In each simulation, the neutron is prepared in an initial state with 41.3% population in the ground state, 24.8% in the first excited state, 24.8% in the second excited state, and 9.1% in the third excited state. This choice disagrees slightly with [3] which describes an exemplary initial state in the experiment as containing 59.6% population in the ground state, 34% in the first excited state, 6.3% in the second excited state and
no population in higher states. However, this initial state gives quite unsatisfactory simula-
tion results compared to data in figures 1 and 2, both in the entropic and conservative
cases. However, simulations for 3 are quite impervious to changes in the initial state.

The neutron state then evolves according to either the conservative (39) or entropic (40)
unitless master equations. Entropic simulations are run with $S$ values of 100, 500 and
$10^3$. After 26 unitless seconds, these simulated neutrons have effectively passed through
the oscillating boundary portion of the Qbounce experiment and the ground state population
is recorded (explanation for this interaction time is detailed in appendix C). Experimental
results for relative transmission (measured transmission divided by transmission with no
oscillations) are then compared with the conservative and entropic simulations in figures 1,
2 and 3.

There is excellent agreement with theoretical predictions in the simulations. As equation
(23) predicts, transmission values for entropic simulations approach those of the conserva-
tive model as $S$ increases. This is to say, classical gravity can be recovered with large
enough $S$ in the entropic model. The goal then becomes to place a lower bound on $S$.

In figures 1, 2 and 3, there is good agreement with experimental data and entropic sim-
ulations when $S$ equals 500 and $10^3$. $S = 100$ appears to be too low as its corresponding
predictions are typically far-off from the data in all three figures. There are some data points
in 1 and 2 not explained by any entropic simulations, namely the transmission probability
at $\omega = 575$ Hz in figure 1 and the point at $b = 4$ mm/s in figure 2. However, these points
are also not explained by the conservative model, so these anomalies pose open questions
in need of further explanation. Still, 500 seems to be an excellent lower bound for $S$.

V. Conclusions and Further Directions

We have shown that gravity acting near Earth’s surface can be modeled as an environ-
ment coupled to neutrons. Despite the compelling criticism outlined in [14], this entropic
gravity model is capable of maintaining both strong coupling and minimal decoherence
and can predict results of ultra-cold neutron experiments [3]. In fact, In the limit where $S$
approaches infinity in the entropic model, conservative gravity is recovered.

This model for entropic gravity is far from complete. By virtue of being a near-Earth
FIG. 1: Transmission probability for varying oscillation frequency when the oscillation strength equals 2.05 mm/s.

FIG. 2: Transmission probability for varying oscillation strength when the boundary frequency \( \omega = \omega_{03} = \frac{E_2 - E_0}{\hbar} \).

approximation tested against one type of experiment featuring one type of matter, it is far from a universal theory. Entropic gravity must also account for results from atom interferometry [4] and Ramsey spectroscopy for neutrons [1]. In particular, now that \( \mathcal{S} \) has been benchmarked at about 500, it would be insightful to see if this value carries over to other experimental results. The model must also be generalizable beyond Earth’s surface and the low-energy regime. Additionally, these entropic predictions are only useful if they are capable of explaining something previously unexplainable such as dark matter. Otherwise, this gravitational theory will continually fall victim to Occam’s razor.

Regardless, our hopes for this model were much more modest. The results presented here should indicate that entropic gravity cannot be discarded for reasons of excess deco-
FIG. 3: Transmission probability for varying oscillation strength when the boundary frequency $\omega = \omega_0 = \frac{E_3 - E_0}{\hbar}$.

herence. Although entropic gravity may fail for other reasons, the goal now becomes to stretch the theory further and seek out these potential criticisms.


A. Solving the Schrödinger Equation For a Bouncing Ball

In this section, we solve the quantum bouncing ball problem (as is done in [16]). Consider the time-independent Schrödinger equation for a particle of mass $m$ experiencing a linear gravitational potential $U(\hat{x}) = mg\hat{x}$ and an infinite potential barrier at $x = 0$. We wish to find the eigenvalues $E$ and eigenvectors $|E\rangle$ such that

$$\hat{H}_c |E\rangle = E |E\rangle,$$  \hspace{1cm} (A1)

$$\hat{H}_c = \frac{\hat{p}^2}{2m} + mg\hat{x}.$$  \hspace{1cm} (A2)

Applying $\langle x|$ to equation (A1), the equation can be rewritten as

$$\left(\frac{d^2}{dx^2} - \frac{2m}{\hbar^2}[mgx - E]\right) \langle x|E\rangle = 0,$$  \hspace{1cm} (A3)

and the infinite potential barrier manifests itself in the boundary condition

$$\langle x = 0|E\rangle = 0.$$  \hspace{1cm} (A4)

It is easy to confirm that the solutions to equation (A3) are given by

$$\langle x|E\rangle = c_1 \text{Ai}(\xi + a) + c_2 \text{Bi}(\xi + a),$$  \hspace{1cm} (A5)

where

$$x_0 = \left(\frac{\hbar^2}{2m^2g}\right)^{1/3},$$  \hspace{1cm} (A6)

$$\xi = x/x_0,$$  \hspace{1cm} (A7)

$$a = -\frac{E}{mgx_0},$$  \hspace{1cm} (A8)

and $c_1, c_2$ are constants, and Ai and Bi are the two linearly-independent solutions to the Airy equation

$$\left(\frac{d^2}{dy^2} - y\right)w(y) = 0, w = \text{Ai}(y), \text{Bi}(y).$$  \hspace{1cm} (A9)

Considering the normalization condition

$$\int_0^\infty |\langle x|E\rangle|^2 dx = 1,$$  \hspace{1cm} (A10)
and the asymptotic expansion of Bi, which tends towards infinity as its argument approaches infinity, [5], we can rule its solutions out as unphysical. Applying (A10) to (A3) with \( c_2 = 0 \), we get our normalization coefficient:

\[
c = c(E) = \left[ x_0 \int_0^\infty d\xi \text{Ai}^2 \left( \xi - \frac{E}{m_g x_0} \right) \right]^{-1/2}
\]

(A11)

Thus, solutions in the coordinate representation are given by

\[
\langle x | E \rangle = \frac{\text{Ai}(\xi + a)}{\left[ x_0 \int_0^\infty d\xi \text{Ai}^2(\xi + a) \right]^{1/2}}
\]

(A12)

Applying the boundary condition (A4) yields eigenvalues \( E_n = -mgx_0a_{n+1} \), where \( n = 0, 1, 2, ... \) and \( a_j \) denotes the \( j \)th zero of Ai. By convention, the energy eigenstates of a system are numbered beginning with zero to signify the ground state, whereas the zeroes of a function are numbered beginning with one, hence the \( n \)th energy state corresponding to the \( (n+1) \)th zero of the Airy function. Corresponding eigenfunctions are given by

\[
\langle x | E_n \rangle = \frac{\text{Ai}(\xi + a_{n+1})}{\left[ x_0 \int_0^\infty d\xi \text{Ai}^2(\xi + a_{n+1}) \right]^{1/2}}
\]

(A13)

The set of eigenvectors \( \{|E_n\} \) forms an orthonormal basis.

B. The Quantum Bouncer with an Oscillating Boundary: Change of Variables

In this section, the Schrödinger equation used to model the QBounce experiment is converted to the reference frame of the oscillating boundary. The following is a reproduction of results from [1]. Consider the 1-D time-dependent Schrödinger equation for a particle with potential energy \( U(\hat{x}) \), along with an infinite potential barrier which oscillates with a frequency \( \omega \) and amplitude \( a \) about the point \( x = 0 \):

\[
i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle
\]

(B1)

where

\[
\hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x}) - \frac{\hbar^2}{4m} \delta'(\hat{x} - a \sin(\omega t))
\]

(B2)
When $\langle x \rangle$ is applied on the left to both sides of (B1), one gets the Schrödinger equation in the coordinate representation:

$$i\hbar \frac{d}{dt} \langle x | \psi(t) \rangle = \left\{ -\frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x^2} + U(\hat{x}) \right\} \langle x | \psi(t) \rangle,$$

$$-\frac{\hbar^2}{4m_i} \delta'(x - a \sin(\omega t)) \right\} \langle x | \psi(t) \rangle.$$  \hspace{1cm} (B3)

Given the infinite potential barrier, one can impose the boundary condition

$$\langle x = a \sin(\omega t) | \psi(t) \rangle = 0.$$  \hspace{1cm} (B4)

The goal is now to convert (B3) to the reference frame of the oscillating mirror. Given the change of variables $\tilde{x} = x - a \sin(\omega t)$, it is easy to show that

$$\frac{\partial^2}{\partial \tilde{x}^2} = \frac{\partial^2}{\partial x^2}, \hspace{1cm} \mbox{B5}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \left( \frac{\partial \tilde{x}}{\partial t} \right) \frac{\partial}{\partial \tilde{x}}, \hspace{1cm} \mbox{B6}$$

$$= \frac{\partial}{\partial t} - a \omega \cos(\omega t) \frac{\partial}{\partial \tilde{x}}.$$  \hspace{1cm} (B7)

Thus, the equation of motion (B3) in the reference frame of the oscillating barrier becomes

$$i\hbar \frac{\partial}{\partial t} \langle \tilde{x} | \tilde{\psi}(t) \rangle = \left\{ \hat{H}_0 + W(\tilde{x}, t) \right\} \langle \tilde{x} | \tilde{\psi}(t) \rangle,$$

where

$$\hat{H}_0 = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \tilde{x}^2} + U(\tilde{x}) - \frac{\hbar^2}{4m} \delta'(\tilde{x}), \hspace{1cm} \mbox{B9}$$

$$W(\tilde{x}, t) = U(a \sin \omega t) + i\hbar a \omega \cos(\omega t) \frac{\partial}{\partial \tilde{x}}, \hspace{1cm} \mbox{B10}$$

$$U(\hat{x} + a \sin \omega t) = U(\hat{x}) + U(a \sin \omega t), \hspace{1cm} \mbox{B11}$$

$$\langle \tilde{x} | \tilde{\psi}(t) \rangle = \langle x | \psi(t) \rangle.$$  \hspace{1cm} (B12)

Notice how when the time-dependent term $W(\tilde{x}, t) = 0$, the equation of motion reduces to the time-independent quantum bouncing ball problem of section A. To simplify notation, return $\tilde{x} \to x$ and $\tilde{\psi}(t) \to \psi(t)$ in equations (B8)-(B10) and rewrite the original Schrödinger equation (B8) from the mirror’s reference frame:

$$i\hbar \frac{\partial}{\partial t} | \psi(t) \rangle = \left\{ \hat{H}_0 + \hat{W} \right\} | \psi(t) \rangle,$$

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + U(\hat{x}) - \frac{\hbar^2}{4m} \delta'(\hat{x}), \hspace{1cm} \mbox{B14}$$

$$\hat{W} = U(a \sin \omega t) - a \omega \cos(\omega t) \hat{p}.$$  \hspace{1cm} (B15)
Note that the $\hat{p}$ operator in this last equation is only present so as to be transformed into $-i\hbar \frac{\partial}{\partial x}$ when $\langle x \rangle$ is reapplied.

When we convert our Schrodinger equation (B13) into the density matrix formalism, we get that

$$
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \hat{H}_0 + \hat{W}, \hat{\rho} \right]
$$

(B16)

and

$$
\frac{d\hat{\rho}}{d\tau} = -\frac{i}{mgx_0} \left[ \hat{\rho}^2 + U(\hat{x}) - \frac{\hbar^2}{4m} \delta'(\hat{x}) - a\omega \cos(\omega t) \hat{p}, \hat{\rho} \right].
$$

(B17)

Note that the time-dependent term $U(a \sin \omega t)$ disappears from the commutator since it has no dependence on an operator.

Furthermore, consider the $\mathcal{D}(\hat{\rho})$ operator in the entropic model given by equation (21). Under the change of variables $\hat{x} = x - a \sin(\omega t)$, $\mathcal{D}(\hat{\rho})$ is invariant under the change of variables since

$$
\exp\left( -\frac{i(\hat{x} + a \sin \omega t)}{x_0 \mathcal{F}} \right) \hat{\rho} \exp\left( +\frac{i(\hat{x} + a \sin \omega t)}{x_0 \mathcal{F}} \right) = \exp\left( -\frac{i\hat{x}}{x_0 \mathcal{F}} \right) \hat{\rho} \exp\left( +\frac{i\hat{x}}{x_0 \mathcal{F}} \right).
$$

(B18)

C. Qbounce Simulation Matrix Elements

In order to simulate the Qbounce experiment using QuTiP [7], the master equations (C1) and (C2) must first be made unitless. This can be accomplished by differentiating with respect to unitless time $\tau = (tmgx_0)/\hbar$. The conservative master equation becomes

$$
\frac{d\hat{\rho}}{d\tau} = -\frac{i}{mgx_0} \left[ \hat{h}^2 + \hat{\xi} + \hat{w}, \hat{\rho} \right]
$$

(C3)

and the entropic Lindblad equation becomes

$$
\frac{d\hat{\rho}}{d\tau} = -\frac{i}{mgx_0} \left[ \hat{\rho}^2 + \hat{\xi} + \hat{w}, \hat{\rho} \right] + \frac{\mathcal{D}(\hat{\rho})}{mgx_0}.
$$

(C2)

Solving for matrix elements by sandwiching these master equations between $\langle E_j \rangle$ on the left and $|E_k\rangle$ on the right yields the unitless conservative master equation

$$
\frac{d\hat{\rho}}{d\tau} = -i \left[ \hat{h} + \hat{\xi} + \hat{w}, \hat{\rho} \right],
$$

(C3)
along with the unitless entropic master equation

\[
\frac{d\hat{\rho}}{d\tau} = -i [\hat{h} + \hat{w}, \hat{\rho}] + \mathcal{F} \left( \mathcal{D}\hat{\rho}\mathcal{D}^\dagger - \hat{\rho} \right),
\]  

(C4)

where

\[
h_{jk} = -a_{j+1} \delta_{jk} - \int_0^\infty d\xi \xi \text{Ai}(\xi + a_{j+1}) \text{Ai}(\xi + a_{k+1}) \frac{N_j N_k}{N_j N_k},
\]  

(C5)

\[
\xi_{jk} = \int_0^\infty d\xi \xi \text{Ai}(\xi + a_{j+1}) \text{Ai}(\xi + a_{k+1}) \frac{N_j N_k}{N_j N_k},
\]  

(C6)

\[
w_{jk} = +i \left( \frac{4m}{\hbar g} \right)^{1/3} (a\omega) \cos(\omega t) \frac{\int_0^\infty d\xi \text{Ai}(\xi + a_{m+1}) \frac{d}{d\xi} \text{Ai}(\xi + a_{n+1}) \frac{N_j N_k}{N_j N_k}}{N_j N_k},
\]  

(C7)

\[
D_{jk} = \frac{\int_0^\infty d\xi \exp(-i\xi/\epsilon) \text{Ai}(\xi + a_{j+1}) \text{Ai}(\xi + a_{k+1}) \frac{N_j N_k}{N_j N_k}}{N_j N_k},
\]  

(C8)

\[
N_l = \left[ \int_0^\infty d\xi \text{Ai}^2(\xi + a_{l+1}) \right]^{1/2}. \]

(C9)

Here, \( \hat{h} \) gives the boundary and kinetic energy term. \( \hat{\xi} = \hat{x}/x_0 \) is the unitless position operator, \( \hat{w} \) accounts for the accelerating frame, \( \mathcal{D} \) is the first exponential term in \( \mathcal{D}(\hat{\rho}) \), and \( N \) is the normalization factor. In a similar fashion to finding the above matrix elements, we can show that the matrix elements for the position and momentum operators \( \hat{x} \) and \( \hat{p} \), along with \( \delta''(\hat{x}) \) in the \( |E_i\rangle \) basis are given by

\[
x_{jk} = x_0 \frac{\int_0^\infty d\xi \xi \text{Ai}(\xi + a_{j+1}) \text{Ai}(\xi + a_{k+1}) \frac{N_j N_k}{N_j N_k}}{N_j N_k},
\]  

(C10)

\[
p_{jk} = -\frac{i\hbar}{x_0} \frac{\int_0^\infty d\xi \text{Ai}(\xi + a_{j+1}) \frac{d}{d\xi} \text{Ai}(\xi + a_{k+1}) \frac{N_j N_k}{N_j N_k}}{N_j N_k},
\]  

(C11)

\[
\delta''_{jk}(\xi) = \frac{\left[ \frac{d}{d\xi} \text{Ai}(\xi + a_{j+1}) \right]_\xi=0 \left[ \frac{d}{d\xi} \text{Ai}(\xi + a_{k+1}) \right]_\xi=0 \frac{N_j N_k}{N_j N_k}}{N_j N_k},
\]  

(C12)

We can thus define the unitless position and momentum operators \( \hat{\xi} \) and \( \hat{\pi} \) as

\[
\hat{\xi} = \hat{x}/x_0,
\]  

(C13)

\[
\hat{\pi} = \frac{\hat{p}x_0}{\hbar}.
\]  

(C14)

Now, the oscillation strength \( b \) appears in the \( \hat{w} \) term in a very non-trivial way. In [3], they define the classical Rabi from the \( p \)th energy state to the \( q \)th oscillation in their experiment as
\[
\Omega_R = \frac{b}{x_0(a_1 - a_{q+1})}
\]  
(C15)

where \(s\) is the oscillation strength and \(a_n\) represents the \(n\)th zero of the Airy function. The classical Rabi oscillation is also defined as

\[
\Omega_R = \frac{(a\omega)}{\hbar} |\langle E_p | \hat{p} | E_q \rangle| = \frac{(a\omega)}{\hbar} p_{pq}.
\]  
(C16)

(C17)

Setting these two equal to each other, we get a relationship between \(a\omega\) and oscillation strength:

\[
a\omega = \frac{\hbar b}{(a_p - a_q)x_0 p_{pq}} = \frac{b}{(a_p - a_q)\pi_{pq}}
\]  
(C18)

(C19)

where \(\pi_{jk}\) represents the \(j\)th row, \(k\)th column entry in the unitless momentum operator (C14). Plugging this result into (C7) gives

\[
w_{jk} = +i \left( \frac{4m}{\hbar g} \right)^{1/3} \frac{b}{(a_p - a_q)\pi_{pq}} \cos(\omega t) \int_0^\infty d\xi \frac{\text{Ai}(\xi + a_{m+1})}{N_j N_k} \frac{d}{d\xi} \text{Ai}(\xi + a_{n+1}).
\]  
(C20)

D. Time of Flight Calculation for \textit{Q}bounce

Determining the time of flight for \textit{Q}bounce simulations is not necessarily clear-cut. According to [3], the horizontal neutron velocities \(v\) are between 5.6 and 9.5 m/s. Furthermore, the length of region II is 20 cm. Thus, the time-of-flight in region II falls within a range:

\[
0.20 \text{ s,} < \Delta t < \frac{0.20}{5.6} \text{ s,}
\]  
(D1)

\[
0.021 < \Delta t < 0.036 \text{ s.}
\]  
(D2)

Converting to unitless time \(\tau\) gives

\[
\Delta \tau = \frac{m_g g x_0}{\hbar} \Delta t = (913.97) \Delta t.
\]  
(D3)

(D4)
Hence,

\[ 19.2 < \Delta \tau < 32.9. \]  \hspace{1cm} \text{(D5)}

We choose \( \tau = 26 \) for simulations since it is an intermediate value in the range above.